CALCULUS.

400. Proposed by H. S. UHLER, Yale University.

The axis of a prism whose right-section is a regular polygon of apothem a and of n sides passes through the center of a sphere of radius R. Show that, in general, the volume may be expressed by the formula:

$$V = \frac{4}{3}\pi R^3 + \frac{2}{3}a^2 n\left(R^2 - a^2\sec^2\frac{\pi}{n}\right)^{1/2}\tan\frac{\pi}{n} + \frac{1}{3}an(3R^2 - a^2)\sin^{-1}\left[\frac{2a\left(R^2 - a^2\sec^2\frac{\pi}{n}\right)^{1/2}\tan\frac{\pi}{n}}{R^2 - a^2}\right]$$
$$- \frac{4}{3}nR^3\sin^{-1}\left[\frac{R\sin\frac{\pi}{n}}{(R^2 - a^2)^{1/2}}\right].$$

Also discuss the two special cases where

$$a = R\cos\frac{\pi}{n}, \text{ and } n = \infty.$$

401. Proposed by LAENAS G. WELD, Pullman, Illinois.

Given a continuum of triangles whose sides are in arithmetical progression, the common difference being h: (a) The ratio of the mean value of the areas of all the triangles, the mean of whose three sides is not greater than μ, to the area of the triangle, the mean of whose three sides is equal to μ, is $(\mu + 2h)/3\mu$. Indicate the limiting values of this ratio and show that, when it is equal to $\frac{1}{2}$, the triangle whose mean side is μ is right angled. (b) The ratio of the mean value of the areas of the circles inscribed in all of these triangles, the mean of whose three sides is not greater than μ, to that of the circle inscribed in the triangle, the mean of whose three sides is equal to μ, has the limiting values $\frac{1}{2}$ and $\frac{1}{3}$. When the triangle whose mean side is μ is right angled, the ratio in question is 4/9. (c) Of the circles circumscribed about these triangles the minimum has the radius $2h$.

MECHANICS.

319. Proposed by LAENAS G. WELD, Pullman, Illinois.

A hexagonal pencil lies upon the inclined top of a drawing table and is on the point of either rolling or sliding. Find the angle between its direction and the horizontal edge of the table, the coefficient of friction being μ.

320. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A heavy uniform chain of length l is hung over a rough horizontal cylinder of radius r. Show that one end of the chain will be

$$\frac{2\mu r}{\mu^2 + 1}(e^{\mu\pi} + 1) + l(e^{\mu\pi} - 1)$$

units lower than the other, just when the chain begins to move, the coefficient of friction being μ.

NUMBER THEORY.

237. Proposed by NORMAN ANNING, Chilliwack, B. C.

Prove that for three numbers, x, y, z,

$$9\Sigma(x - y)^4 = \Sigma(2x - y - z) = 2\square.$$

238. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Determine the rational value of x that will render $x^3 + px^2 + qx + r$ a perfect cube. Apply the result to $x^3 - 8x^2 + 12x - 6$.